

Strings as Physical Lines in Vacuum: Basic Concepts for the Computation of the Planck Length and the Planck Mass

Aharon Nudelman*

P.O. Box 7377
Ashkelon, 78172
Israel

Abstract

Certain linear objects, termed physical lines, are considered, and initial assumptions concerning their properties are introduced. A closed physical line in the form of a circle, termed *J-string*, is singled out for investigation. It is shown that this curve consists of indivisible line segments of length ℓ_Δ . It is assumed that a *J-string* has an angular momentum whose value is \hbar . It is then established that a *J-string* of radius R possesses a mass m_J , equal to $\hbar/2\pi cR$, a corresponding energy, as well as a charge q_J , where $q_J = (\hbar c/2\pi)^{1/2}$. It is also established that $\ell_\Delta = 2\pi(\hbar G/c^3)^{1/2}$, where c is the speed of light and G is the gravitational constant. Based upon investigation of the properties and characteristics of *J-strings*, a method is developed for the computation of the Planck length and mass (ℓ_P^*, m_P^*) . The values of ℓ_P^* and m_P^* are computed according to the resulting formulae (and given in the paper); these values differ from the currently accepted ones.

1. Introduction

1.1. String theory is one of the most active areas of research in modern physics at the present time. Major contributions to this field were made by Green, Polchinski, Schwartz, Witten and others, see [2, 5, 6] and references therein. In the present work, we limit our consideration to certain inherently physical aspects of string theory. In particular, we introduce a certain new approach to the investigation of strings. In formulating this approach, we have made use, to some extent, of the well-known model of de Broglie introduced in 1923 — namely, a string in the form of a circle that may be characterized in the four-dimensional space-time.

*This work was carried out in part while the author was a doctoral student and a senior researcher at the CSREP Research Institute, Moscow, Russia.

1.2. Among the range of questions and problems that arise in string theory, one can single out those dealing with the Planck length and the Planck mass. The formulae currently in use for the computation of these fundamental quantities (as well as the Planck time) were derived by Planck back in 1899, by means of an analysis of the units of measure of three primary constants: h , c , and G . However, the “natural” (objective) physical units introduced by Planck have gained widespread notoriety only much later (it was not before the 1950’s that there was active interest in the Planck length; see, for example [7]). As is well known, questions concerning the Planck length constitute one of the key problems in string theory today (see the lecture notes [4, pp. 5–7], for example). Thus one cannot accept as satisfactory a situation where the values of these two fundamental physical quantities are computed solely from the analysis of units of measure of certain constants. In this work, based upon the aforementioned approach to the study of strings, we propose an alternative method for the computation of the Planck length and the Planck mass, and develop the corresponding formulae. Since this article is intended as a discussion item, certain key issues and concepts are described in some detail. The article is subdivided into individually numbered paragraphs (we use this numbering for internal reference).

2. Initial assumptions

2.1.1. To quote [3], a fundamental postulate of the special theory of relativity is formulated as follows:

Light is always propagated in empty space with a definite speed c that is independent of the state of motion of the emitting body.

The foregoing postulate, in essence, consists of two independent assertions:

1. Light always has a certain nonzero velocity. This means that the quanta of light (photons) are always — that is, in any frame of reference — in motion.
2. The value v of the velocity of light (photons) is always — that is, in any frame of reference — constant and equal to c .

2.1.2. It is inherent to this postulate that the first assertion above does not require indication of a frame of reference relative to which the motion of photons is considered. Consequently, there exist physical objects for which the assertion that they are in motion is true without specifying an observation framework, that is, without choosing a definite frame of reference.

2.1.3. For any physical object moving at a certain velocity v , the dimension L_v along the direction of its motion, according to Lorentz transformations, is given by

$$L_v = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

where L_0 is the dimension of this object at rest. If $v \rightarrow c$, then $L_v \rightarrow 0$. Let us adopt the following assumption: at $v = c$, the value of L_v may be actually equal to zero.

2.1.4. Let us introduce a postulate, termed the *VL-postulate*, that relates ultimate velocity to vanishing (zero) dimension.

If the dimension of a physical object is equal to zero along a given straight line (direction), then — assuming conservation of energy of the object — this is necessary and sufficient for this object to move in vacuum at an ultimate velocity along the given line (direction).

2.2.1. Let us assume the following: vacuum is an aggregate of individual *physical points*. Using the general approach of Cantor [1] to the description of “aggregates” (manifolds) of mathematical points, physical points can be defined as objects of zero energy that have zero dimension in every direction.

2.2.2. For an individual physical point, it becomes necessary to observe that, according to the VL-postulate, it is always in motion at the ultimate velocity simultaneously in all directions. This means that no displacement of the physical point takes place.

2.2.3. For future reference to the foregoing notion of vacuum, let us introduce the terminology *physical vacuum* to denote a vacuum as it is defined in §2.2.1 and §2.2.2.

2.3.1. Let us suppose that physical vacuum may contain, in addition to individual and independent physical points, also associations of such points that are bound together in some manner. Let us assume that the foregoing associations of points are characterized by local continuity and are always linear. We call such associations of points *physical lines*. Let us also assume that there exist physical lines that are characterized by both local continuity and local curvature; we call this type of physical lines *physical curves*.

2.3.2. A continuous physical line of a certain length L cannot be partitioned, “dissected,” into separate physical points: any part of this line, as small as desired, is still a line segment and not a point. Let us assume that there has to exist a special physical object — a certain small line segment that is indivisible into parts.

2.3.3. Let us, for the time being, denote the length of this object by z_Δ . We call this object a *physical line-element*, and define it as follows.

A physical line-element is a continuous linear association of physical points, indivisible into parts, that has length z_Δ and zero cross-section.

Let us assume that the properties of this physical object can be characterized in two ways. The object can be interpreted as a single indivisible conglomerate of physical points and, at the same time, as a free assemblage thereof.

2.3.4. A physical line of length L , as well as any other line, can be either continuous throughout or discrete — that is, constructed of continuous segments and gaps between them. The length l of any segment (or gap) must be an integer multiple of z_Δ .

2.4.1. Let us introduce the following working hypothesis:

In the observable Universe, there exists nothing except physical vacuum and physical lines.

2.4.2. Let us point out the main consequence of the foregoing hypothesis.

All the energy and mass in the manifest Universe are concentrated in the form of physical lines.

3. Planar physical lines of constant curvature and their properties

3.1.1. Consider the following hypothetical object: a special continuous closed physical curve in the form of a circle. Let us call this physical curve a *J-string*. We adopt the following postulate that may be regarded as the definition of a *J-string*.

A *J-string* is a special continuous closed physical curve in the form of a circle that constitutes an association of physical line-elements and, at the same time, the trajectory along which these line-elements are moving so that the angular momentum of a *J-string* is equal to \hbar .

3.1.2. Given a physical curve, we denote the length, as measured along the arc, of a constituent physical line-element by ℓ_Δ . Let us now investigate the physical properties of a line-element of length ℓ_Δ and of a *J-string*. Consider an arbitrary circle. Let us distinguish a small segment of length ΔL on the circle, and draw a tangent through one of its points. The dimension of the segment along this tangent is equal to the length its projection onto the tangent.

By definition, a *J-string* is an association of physical line-elements jointly comprising a circle. Let us now draw tangents to one of such line-element — a circular segment of length ℓ_Δ . Assume, hypothetically, that tangents are drawn through each point of this line-element. It would appear that the dimension of the line-element along any of these tangents must be also equal to the length of its projection onto the corresponding tangent.

According to §3.1.1, all the line-elements of a *J-string* are moving in a circular trajectory, which coincides with the *J-string* itself. The linear velocity v , at each point of any such line-element of length ℓ_Δ , is directed along a tangent to the trajectory. One can assert the following: any such line-element is moving with linear velocity v simultaneously along all tangents to the element.

At a given moment t , each of the points of the line-element is moving, with velocity v , solely along the tangent corresponding to this point: point A is moving along the tangent at A , point B is moving along the tangent at B , and so forth. It follows that at time t , the dimension of the line-element along any given tangent (for example, along the tangent at A) is equal to the “length” of the point of tangency (point A itself) — that is, equal to zero. Invoking the VL-postulate, we come to the conclusion that the linear velocity v along any tangent to the considered line-element must necessarily be ultimate: $v = c$. The foregoing analysis applies to each of the line-elements comprising a *J-string*; hence, it also holds for the *J-string* as a whole.

A comparative physical model. Consider a closed non-elastic (“soft”) thread of a certain mass that has an arbitrary shape and lies on a planar horizontal surface. Suppose that the surface and the thread are rotated in such a manner that the thread takes the shape of a ring of radius R , uniformly distended along all of its length by a radial centrifugal force (assume that the ring is rotating in its plane about its geometric center). Let us think of the rotating ring as a torus, and let us call the circular line centrally located within the torus the *circular axis of the ring*. The linear velocity of rotation is then directed along a tangent to the circular axis of the ring (torus) and has some arbitrary value v . The diameter of the cross-section of the ring (torus) is equal to d , where $d \ll R$.

Each small segment of the ring will have a certain nonzero dimension along the direction of its velocity — that is, a certain non-vanishing projection onto any tangent to the circular axis of the ring (torus). The length of the projection, onto tangent A for example, is equal to the length (or part of the length) of the long diameter of the ellipse that arises as the cross-section of the ring (torus) by a plane perpendicular to the horizontal surface and passing through the point of

tangency A . Now suppose that $d \rightarrow 0$. The physical situation described above remains the same. However, if d were actually equal to 0, the situation would have been fundamentally different. The ellipse obtained by the aforementioned cross-section of the ring collapses into a point (the point of tangency). In this case — according to the VL-postulate — the linear velocity v is no longer arbitrary, it must necessarily have the ultimate value.

3.2.1. Let us draw, through each point of the physical line-element considered in §3.1.2, a plane perpendicular to the tangent at that point. The size of the cross-section of a physical line by such a plane is clearly zero. It follows that each line-element of a J -string, according to the VL-postulate, must move — along certain directions — in the plane of its cross-section, at the ultimate velocity. In order to find these directions, let us again consider the physical line-element of length ℓ_Δ and draw tangents through all of its points. In addition to the tangents, let us draw through each point of the line-element two mutually perpendicular straight lines (normals) that lie in the plane of the cross-section. Thus at any point of the line-element, a tangent and the two normals form three orthogonal axes.

3.2.2. Let us assume that one of the two normals lies in the plane of the J -string that contains the considered line-element. This normal will be directed radially, either from the center or towards the center of the J -string. According to the VL-postulate, all the points of the considered line-element (and, hence, the entire element) must move at the ultimate velocity simultaneously along all radial directions. Such simultaneous motion of the line-elements in radial directions corresponds to a concentric expansion (or concentric contraction) of the entire J -string, while the geometric center of the J -string remains at rest. Thus we have a resting J -string that expands (or contracts) radially with velocity c . Let us call such a state of the J -string *state* α .

3.2.3. If the first of the two normals under consideration lies in the plane of the J -string, then the second normal is perpendicular to this plane. Each point of the physical line-element under consideration and, hence, the entire element (as well as all the other elements of the J -string), must necessarily move at the ultimate velocity along the second normal, in accordance with the VL-postulate. This means that the geometric center of the J -string must move at the ultimate velocity ($v = c$) along an axis perpendicular to the plane of the J -string. (Along such an axis, not only every line-element of the circle, but also the J -string as a whole, has dimension zero.) Thus we have a J -string that moves along a line perpendicular to its plane with velocity c . Let us call such a state of the J -string *state* β .

3.2.4. Consider again the line-element of §3.1.2. Let us fix an arbitrary plane of its cross-section and draw an arbitrary straight line in this plane. Assume that this line passes through the section point and is inclined at an angle θ , where $0 < \theta < \pi/2$, to the plane of the J -string. Furthermore, let us draw lines parallel to the line described above through all the points of the line-element.

In our analysis of states α and β , we have considered the motion of each point of the physical line-element — at velocity c , along a given axis — as being independent from the motion of all other points, as long as the integrity of the line-element was maintained (such analysis is based upon the properties of a physical line-element, described in §2.3.3). However, the case considered here is different from the two cases described in the foregoing paragraphs. Any specific point of the line-element and, hence, the element as a whole — as a separate entity — can move at the ultimate velocity, at least in principle, along an arbitrarily straight line (and lines parallel to this straight line), such as the one(s) described above. But for a J -string as a whole such motion is impossible. Unlike a physical line-element, which by definition (cf. §2.3.3) is an association of

points, a J -string is an association of line segments, that is, physical line-elements of length ℓ_Δ . Hence a J -string always has a certain nonzero dimension along a line inclined at an angle θ to its plane — the projection of a J -string, as a whole, onto any such line is not equal to zero.

It follows that in this physical situation, the VL-postulate does not apply. Consequently, the two assertions of the VL-postulate, about the necessity of motion and about the ultimate velocity of motion, do not extend to the case considered here. In this work, we assume that motion of a J -string with velocity v , where $0 \leq v < c$, along an axis inclined at an arbitrary angle $\theta \neq \pi/2$ to its plane is possible but not necessary. We say that a J -string in such a motion is in **state** γ .

3.2.5. Assume that if certain conditions arise, a J -string can transit from one state to another.

3.2.6. In addition to the analysis carried out in §3.1.2–§3.2.3, it is essential to point out the following with respect to the motion at ultimate velocity considered in these paragraphs. For example, as shown in §3.2.3, a J -string in state β must move at velocity $v = c$ along an axis perpendicular to its plane. The motion along this axis can occur in any of two opposite directions. If there is no “preferable direction” (that is, if the two directions are completely equivalent — there is nothing that may distinguish one direction relative to the other), then an actual displacement of the J -string along this axis is impossible. According to the VL-postulate, the J -string will move at ultimate velocity simultaneously in two opposite directions (see, for comparison, §2.2.2). This condition — that there exist a “preferable direction” for an actual displacement to occur — clearly extends to the circular rotation of a J -string, considered in §3.1.2, and also to the potential expansion or contraction of a J -string in state α , considered in §3.2.2. In this regard, we point out the following:

For state α : Conditions for the realization by a J -string of its potential concentric expansion or contraction are outlined in Section 5 of this paper (see §5.3.2).

For state β : Investigation of the physical factors that may cause a J -string to move in one of the two directions along an axis perpendicular to its plane is beyond the scope of this work.

For circular rotation:

- The movement (displacement) of the line-elements comprising a J -string along a circular trajectory is a fundamental physical property of the J -string; this property follows directly from the condition that a J -string has to have an angular momentum (see the definition in §3.1.1).
- Any physical object has an angular momentum if and only if a preferable direction of rotation is given in some manner.

4. Parameters and characteristics of planar physical lines of constant curvature

4.1.1. Let us consider a J -string of radius R in state γ . Suppose that this J -string is moving (actual displacement takes place) in a certain frame of reference at velocity v , where $v \rightarrow 0$. Let us now assume that the value of v becomes zero. By definition, any J -string is a continuous physical line in the form of a circle. If $v = 0$, then a single parameter is necessary and sufficient to completely characterize the J -object: its radius R . Based on §2.4.2, let us assume that the

J -string must have a mass, which we denote by m_J . It follows that the value of m_J depends *only* on the radius of the J -string, that is $m_J = f(R)$.

4.1.2. In §3.1.2, we have considered, as a comparative physical model, a thread whose cross-section has diameter $d \neq 0$. This object, as is well known, consists of corpuscles (atoms and molecules) bound together in some manner. Let us call this and similar objects *corpuscular lines*.

Suppose that a corpuscular line of mass m , in the shape of a circle of radius R , rotates in its plane about its geometric center. Let T denote the magnitude of the angular momentum of this corpuscular line. It is well known that

$$T = mvR$$

where v is the magnitude of the linear velocity of rotation, directed along a tangent to the circle. Let us assume that this expression also holds for a J -string of radius R . As shown in §3.1.2 above, for a J -string we have $v = c$, and consequently

$$T_J = m_J c R \quad (1)$$

According to §3.1.1, we have

$$T_J = \frac{h}{2\pi} \quad (2)$$

From (1) and (2), we have

$$m_J c R = \frac{h}{2\pi} \quad (3)$$

Thus we arrive at the following expression for the function $f(R)$ that was introduced in the foregoing paragraph:

$$m_J = \frac{h}{2\pi c} \frac{1}{R} \quad (4)$$

4.1.3. The foregoing relation between the mass of a J -string and its radius may be also deduced in an alternative manner, using special theory of relativity. Assume that the J -string in state γ is moving (actual displacement takes place) in its own plane with velocity v , while maintaining the shape of a circle; the velocity vector lies in the plane of the J -string, and $0 \leq v < c$. One can write-down the following well-known relations for the parameters of this J -string:

$$R_v = R_0 \sqrt{1 - \frac{v^2}{c^2}} \quad \text{and} \quad m_{J,v} = m_{J,0} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

from which it follows that

$$\frac{R_v}{R_0} = \sqrt{1 - \frac{v^2}{c^2}} = \frac{m_{J,0}}{m_{J,v}}$$

Since $m_J = f(R)$ as shown in §4.1.1, we have $R_v/R_0 = f(R_0)/f(R_v)$. This means that for any value of the velocity v , the following relation holds:

$$R_0 f(R_0) = R_v f(R_v) = \text{const}$$

Denoting the value of the constant above by b , we obtain the expression

$$f(R) = b \frac{1}{R} = m_J \quad (5)$$

where b must be equal to $h/2\pi c$ according to (4).

4.2.1. As is well known, if a material point is moving in a circular trajectory, then the frequency of its oscillations with respect to any axis passing through a diameter of the trajectory is identically equal to the frequency of its revolutions along the circumference. Let us introduce the concept of *frequency of a J -string* and denote it by ν_J . Let us adopt the following expression for the frequency of a J -string:

$$\nu_J = \frac{c}{2\pi R} \quad (6)$$

The parameter ν_J reflects the rotation of a J -string in its plane, that is, the motion of each of its line-elements with velocity c along the circle as a trajectory (as postulated in §3.1.1).

According to §2.4.2, a J -string must have not only mass m_J , but also *energy*. Starting with (3), we obtain the following relation

$$m_J c^2 R = \frac{hc}{2\pi} \quad (7)$$

It now follows from (6) and (7) that

$$m_J c^2 = h\nu_J \quad (8)$$

Suppose that a J -string has energy associated with its rotation in its own plane. Let us denote this energy by K_J . We adopt the following assumption: the expression $h\nu_J$ can be interpreted as a formula describing the energy K_J , that is

$$K_J = h\nu_J \quad (9)$$

Then, in view of (8), we have

$$K_J = m_J c^2 \quad (10)$$

Let us *formally* regard the energy K_J as the *kinetic energy of a J -string*.

4.2.2. The J -string under investigation in the foregoing paragraphs, according to §4.1.1, is in state γ and its velocity is assumed to be zero. Assume, hypothetically, that for a short period of time Δt the motion of the physical line-elements along the circumference of the J -string has ceased. In this (completely hypothetical) case, during the time Δt , the J -string would be a static line in the form of a circle. Such an object, in principle, cannot have kinetic energy. However, a static physical line in the form of a circle still embodies a certain type of geometric information, namely a certain well-defined order of a part of physical vacuum. Let us assume that a J -string has, in addition to the energy K_J associated with its rotation, also another kind of energy — the energy U_J associated solely with the shape and radius of its physical line. Let us *formally* regard the energy U_J as the *potential energy of a J -string*.

5. J -strings

5.1.1. Let us continue the investigation of a J -string of radius R and mass m_J in state γ , under the condition that $v = 0$. This physical object is a corporeal line of certain mass that has the form of a circle and rotates in its plane. For all known objects of this type (for example, for the corpuscular line considered in §4.1.2), each small line segment ΔL is subject to a centrifugal force $(\Delta L/2\pi R)(mv^2/R)$. Let us assume that the corresponding centrifugal force is present in a J -string as well, and that its value can be analogously described as $(\ell_\Delta/2\pi R)(m_J c^2/R)$. Let us introduce a force F_c , regarded as a certain *aggregate centrifugal force*, whose value is given by

$$F_c = m_J \frac{c^2}{R} \quad (11)$$

5.1.2. If F_c were the only force acting on a J -string, it would tear apart. However, in this work we assume that a J -string is stable. Therefore, it is necessary to further assume the existence of a *centripetal force* that is equal to F_c in its magnitude and opposite to F_c in its direction: this force is directed along the radius, towards the geometric center of a J -string. Let us denote the centripetal force by F_J . Then, in view of (11), we have

$$F_J = F_c = \frac{m_J c^2}{R} \quad (12)$$

(Here, F_J and F_c denote the magnitudes of the two corresponding aggregate forces.) One can assume that the energy U_J , introduced in §4.2.2, is associated with the action of the force F_J .

5.2.1. In §3.1.2 we have introduced and studied, as a comparative physical model, a corpuscular line in the shape of a circle that is rotating in its plane with velocity v . The corpuscular line can be formally considered as a “string” (elastic cord) that is uniformly distended by a centrifugal force along its perimeter. Such a “string” is at rest — there is no oscillation (or rotation) of its line of circumference. This interpretation of a rotating corporeal line is fully applicable to the object under investigation, a physical curve of mass m_J rotating in its own plane, as well.

However, there is an essential difference between the two linear objects. The corporeal object described above is a corpuscular line in the shape of a circle rotating in its plane; at the same time, this object can be *formally* considered as a “string” at rest (distended by a centrifugal force). In the present work, we assume that the object under investigation has the following special property. It is, equivalently

- a line in the form of a circle of radius R , rotating in its plane with velocity c ;
- and
- a “resting string” of the same form, distended by a force whose value is $m_J c^2 / R$ (the force is directed along the radii, away from the center).

The foregoing property of a J -string means that it is endowed with a *physical dualism*.

It manifests itself as two fundamentally different objects: a *kinematic object* (a “rotating line”) or a *static object* (a “resting string”). Consequently, all the features and characteristics of a given J -string can be determined in their entirety based on either of the two models: the kinematic model (used earlier in Sections 3 and 4) or the static one.

5.2.2. As mentioned in §3.1.1 and §4.1.2, the object under investigation has an angular momentum T_J . This characteristic belongs solely to the kinematic model of a “rotating line.” A static model of a “resting string” cannot have angular momentum. However, from the concept of the physical dualism of a J -string, introduced above, it follows that the static model must have a certain physical characteristic that is strictly equivalent to T_J .

5.2.3. Let us introduce the notion of *charge of a J -string*, defined as follows:

The charge of a J -string is a dynamical characteristic of this object, regarded as a “resting string,” that is equivalent to its kinematic characteristic — the angular momentum.

Let us denote the charge of a J -string as q_J . As is well known, the speed of light in vacuum c may be also regarded as the electrodynamic constant; let us assume that the *square of q_J is equal to T_J multiplied by the electrodynamic constant*.

5.2.4. Thus we have

$$q_J^2 = cT_J \quad (13)$$

Next, in view of (2) and (13), we arrive at the following expression

$$q_J = \left(\frac{hc}{2\pi} \right)^{\frac{1}{2}} \quad (14)$$

For a J -string of radius R , according to (7), we have

$$q_J^2 = m_J c^2 R \quad (15)$$

Comparing the above expression for q_J^2 to (12), we conclude that the force F_J is given by

$$F_J = \frac{q_J^2}{R^2} \quad (16)$$

5.3.1. Taking into account (14), the expression (16) for the force F_J can be rewritten as follows:

$$F_J = \frac{hc}{2\pi} \frac{1}{R^2} \quad (17)$$

(The same relation holds for the force F_c as well.) It is well known that one of the expressions for the constant of fine structure is $2\pi e^2/hc$, where e denotes the elementary electrical charge. The inverse expression $hc/2\pi e^2$, according to existing computations based on available experimental data, evaluates to about 137 (more precisely, 137.036...) in vacuum. Thus by (17) we have

$$F_J \cong 137 \frac{e^2}{R^2} = 137 F_e$$

where F_e is the magnitude of the force of interaction between two elementary electrical charges whose geometric centers are located at distance R from each other. Consequently, we have

$$q_J^2 \cong 137 e^2$$

5.3.2. Suppose we have a J -string in state α , as described in §3.2.2. In state α —according to the VL-postulate— a J -string must either concentrically expand or concentrically contract at ultimate velocity. Concentric contraction of an isolated J -string is impossible. An isolated J -string of radius R and mass m_J —that is equal, in view of (5), to b/R —cannot give rise to a J -string of a smaller radius $R - \Delta R$ and correspondingly higher mass $b/(R - \Delta R)$. Consequently, an isolated J -string can (and does) only expand with velocity c (in-depth investigation of the concentric expansion of a J -string is beyond the scope of the present work).

6. Quantization in J -strings

6.1.1. According to §2.3.3 and §3.1.2, a physical line-element is an object of length ℓ_Δ that is indivisible into parts. In this work, it is assumed that ℓ_Δ constitutes a certain fundamental length. Let us determine its value. In particular, let us consider the possibility of identifying ℓ_Δ with a well-known physical quantity — the Plank length ℓ_P .

6.1.2. Let us investigate the statement: *In quantum (discrete) space, any linear dimension is an integer multiple of the fundamental length ℓ_P .* Let us proceed with the following analysis. Consider, for example, a circle of radius R . If R is an integer multiple of a certain minimal possible length A , then $R = nA$ while the length L of the circle itself is given by $2\pi(nA) = n(2\pi A)$. This means that L is an integer multiple of another minimal length, say B . The ratio of the two indivisible “units of length” is $B/A = 2\pi/k$, where k is an arbitrary positive integer (e.g., $k = 1$) or a simple fraction. This ratio is irrational, and hence the lengths B and A , in principle, cannot be co-measurable. In other words, these lengths constitute completely independent linear parameters. This leads to the conclusion that there should exist, not one, but *two fundamental lengths*.

6.1.3. Among the range of problems in string theory, the questions about the Planck length stand out as one of the key issues in the field. It is well known that the value of the Planck length is computed according to the formula $(\hbar G/c^3)^{1/2}$ and is equal to $1.616 \cdot 10^{-35}$ m. At the present time, the assumption that ℓ_P is a fundamental length, which determines the dimensions of a quantum of space, is widely accepted. The foregoing analysis in §6.1.2 indicates that it would be necessary to consider, not one, but two physical quantities associated with length: say $(\ell_P^*)_1$ and $(\ell_P^*)_2$; let us assume that $(\ell_P^*)_1 = A$ and $(\ell_P^*)_2 = B$.

6.1.4. Let us extend the problem posed in §6.1.1. The length of an element of the physical line in the form of a circle (that is, ℓ_Δ) is, according to §6.1.2, the fundamental length B . Thus we need to determine not only the value of ℓ_Δ , but also the value of the fundamental length associated with the radius of the circle — that is, the length A . The lengths A and B must be described by expressions that differ from each other in some way; let us assume that this difference (or one of the differences) consists of the absence of the factor of 2π in the expression for the length A .

6.2.1. To determine the general form of the expressions for A and B , we will employ certain assumptions concerning the gravitational constant G , discussed later in this section. First, let us write the well-known expression for the interaction of the electrical charges of an electron and a positron at rest (we assume that their centers of mass are at distance L from each other) in a form that is formally analogous to Newton’s law, namely

$$\frac{e \cdot e}{L^2} = \frac{(\sqrt{\varphi_0} m_0) \cdot (\sqrt{\varphi_0} m_0)}{L^2} = \varphi_0 \frac{m_0 \cdot m_0}{L^2}$$

where $\varphi_0 = (e/m_0)^2$ and m_0 is the mass of each of the two particles, assumed to be at rest in some frame of reference. Let us compare this to Newton’s law for the interaction of the masses of these same particles, namely $G(m_0 \cdot m_0)/L^2$. As is well known

$$G \ll \left(\frac{e}{m_0} \right)^2 \quad \text{that is, } \varphi_0 \gg G$$

Let us now set both particles in motion at velocity v , in such a way that the distance L between them remains constant. Let φ denote the square of the ratio of the charge of each particle to its mass. When v approaches c , the mass m_v of each of the two particles continuously increases while the value φ , correspondingly, decreases (since $e = \text{const}$). Suppose that φ cannot decrease below G . Consequently, there has to exist a certain *limiting value* (let us denote it by φ_z), to which φ , that is, the square of the ratio of the charge of a particle to its mass, must converge for $v \rightarrow c$. Let us write this as follows

$$\lim_{v \rightarrow c} \varphi = \varphi_z \tag{18}$$

where $\varphi_z = pG$ for some $p \geq 1$. The above expressions are fully applicable to an individual, isolated, electron (or positron) as well. If this isolated particle is at rest in a certain frame of reference, then $\varphi_0 = (e/m_0)^2$, while if the particle is moving at $v \rightarrow c$, then, in view of (18),

$$\lim_{v \rightarrow c} \left(\frac{e}{m_v} \right)^2 = pG \quad (19)$$

6.2.2. Let us now consider J -strings. Assume that a J -string of radius R is in state γ and that $v = 0$. Such a J -string has charge q_J and mass m_J , and we can write $\varphi_0^* = (q_J/m_J)^2$. Let us set the J -string in motion at velocity $v \rightarrow c$, in such a way that the velocity vector lies in the plane of the J -string. As shown in §4.1.3, the mass of such an object continuously increases with its velocity. Since the charge q_J of a J -string is constant, it follows that when $v \rightarrow c$ the square of the charge-to-mass ratio—that is, the value φ^* —continuously decreases. One may assume that in this case as well (that is, for a J -string) there exists a certain limiting value φ_z^* . Let us further assume that $\varphi_z^* = p^*G$, where $p^* \geq 1$, and write this as

$$\lim_{v \rightarrow c} \varphi^* = \varphi_z^*$$

6.2.3. Consider a sequence of J -strings with decreasing radii $R_1 > R_2 > R_3 > \dots$. The length of the circumference of a J -string of radius R is $2\pi R$. Let n_J be the total number of line-elements in a J -string. Thus $2\pi R = n_J \ell_\Delta$. The minimal possible value of the length $2\pi R$ must correspond to $n_J = 1$. Let us denote the radius of such a circumference by r_Δ . Explicitly, when $n_J = 1$, we have $R = r_\Delta$ and $2\pi R = \ell_\Delta$. For the foregoing sequence of J -strings of decreasing radii, assuming that the number of elements in this sequence is unbounded, we have $R_i \rightarrow r_\Delta$ in the limit.

According to (5), the radius of a J -string and its mass are related to each other in inverse proportion. Thus if there exists a limiting value for the radius of a J -string, the maximum possible mass of a J -string is also limited. Let us denote this mass by m_Δ , and observe that, as the mass of a J -string increases, the limit of the ratio of its charge to its mass is q_J/m_Δ . Therefore, we conclude that the limiting value φ_z^* introduced in §6.2.2 can be described by the following expression

$$\varphi_z^* = \left(\frac{q_J}{m_\Delta} \right)^2 = p^*G \quad (20)$$

6.2.4. Let us assume that $p^* = 2\pi$ in (20) (this is the only way to satisfy the condition introduced in §6.1.4). Consequently

$$\left(\frac{q_J}{m_\Delta} \right)^2 = 2\pi G \quad (21)$$

Based upon (21), the maximal mass m_Δ can be computed using (14) as follows:

$$m_\Delta = \frac{q_J}{\sqrt{2\pi G}} = \frac{1}{2\pi} \left(\frac{hc}{G} \right)^{\frac{1}{2}} \quad (22)$$

(The general form of the foregoing expression for m_Δ indicates that the maximal mass m_Δ constitutes one of the “Planck units;” it is assumed that m_Δ can be regarded as the Planck mass m_P^* .) From (22) and (4), we can now determine the value of r_Δ , that is, the fundamental length A . Namely:

$$A = r_\Delta = \left(\frac{hG}{c^3} \right)^{\frac{1}{2}} \quad (23)$$

The fundamental length B is, correspondingly, given by

$$B = \ell_{\Delta} = 2\pi \left(\frac{hG}{c^3} \right)^{\frac{1}{2}} \quad (24)$$

The values of the physical quantities computed above are:

$$\begin{aligned} m_{\Delta} &= m_P^* = 8.6838 \cdot 10^{-9} \text{ kg} \\ r_{\Delta} &= (\ell_P^*)_1 = 4.0507 \cdot 10^{-35} \text{ m} \\ \ell_{\Delta} &= (\ell_P^*)_2 = 2.5486 \cdot 10^{-34} \text{ m} \end{aligned}$$

7. Conclusions

7.1.1. This article presents a series of related physical concepts and ideas. We have introduced and studied certain linear objects — physical lines, which have been interpreted as strings. A closed planar line of constant curvature, that is, a physical curve in the form of a circle, was singled out for investigation. The term J -string was introduced to denote this physical curve.

7.1.2. It was assumed that a J -string has an angular momentum of \hbar . It was then demonstrated that the J -string rotates in its plane about its geometric center with linear velocity c .

7.1.3. It was established that a J -string can exist in three distinct states α , β , and γ ; under certain conditions, it may transit from one state to another. In state α , a J -string is expanding (contracting) concentrically with velocity c , while its geometric center is at rest. In state β , a J -string is moving at velocity c along an axis that is perpendicular to its plane, while in state γ it can move (displace) at any velocity v , provided $0 \leq v < c$.

7.1.4. Physical properties and characteristics of a J -string of radius R in state γ have been investigated. It was shown that such a J -string (as well as all J -strings) possesses a mass, an energy, and a charge q_j . The mass m_j of a J -string can be expressed as b/R , where $b = h/2\pi c$. The charge q_j is equal to $(hc/2\pi)^{1/2}$. The circumference line of a J -string is uniformly distended along its entire length by a force directed along the radii, whose absolute value is $hc/2\pi R^2$.

7.2.1. It was assumed that the circumference line of a J -string is composed of elements of a certain length. It was established that the minimal possible radius of a J -string can be expressed as $(hG/c^3)^{1/2}$, while the length of its line-element is given by $2\pi(hG/c^3)^{1/2}$.

7.2.2. Various aspects of the problem concerning the Planck length have been studied. As is well known, the Planck length is regarded in string theory as a fundamental length, which determines the dimensions of a quantum of space.

7.2.3. Based upon investigation of the properties and characteristics of J -strings, a qualitatively new method for the computation of the fundamental length was developed. It was shown that one has to consider, not one, but two primary linear parameters: $(\ell_P^*)_1 = r_\Delta$ and $(\ell_P^*)_1 = \ell_\Delta$.

7.2.4. The problem of the “maximal mass” has been investigated. It is well known that the expression $(hc/2\pi G)^{1/2}$ serves as the basis for the computation of the Planck mass m_P — a certain “maximal mass.” It is widely assumed in the literature that this mass can be correlated with the minimal volume ℓ_P^3 . Thus the expression m_P/ℓ_P^3 is usually invoked in estimating the value of the ultimate density (this parameter plays a fundamental role in the theories of gravity currently being developed).

The question of the existence of a “maximal mass” is extremely important. We have obtained an expression for the mass of a hypothetical object (a J -string), as well as a formula for the computation of the maximal (maximum possible) mass of an elementary object — the mass m_Δ . Specifically $m_\Delta = \frac{1}{2\pi}(hc/G)^{1/2}$ (it is assumed that m_Δ is, in fact, the mass m_P^*).

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